

Quantization of Discretized Spacetimes and the Correspondence Principle

Ioannis Raptis¹ and Roman R. Zapatrin²

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An algebraic quantization procedure for discretized spacetime models is suggested based on the duality between finitary substitutes and their incidence algebras. The provided limiting procedure that yields conventional manifold characteristics of spacetime structures is interpreted in the quantum algebraic framework as a correspondence principle.

MOTIVATION

Current physical theory predicts that at small scales the conventional picture of spacetime as a 4-dimensional differential manifold breaks down to something more discrete, finitary, and quantum. This inadequacy of the smooth spacetime manifold is due on one hand to the ideal character of event determinations of a classical observer, on the other to the appearance of singularities.

To deal with the first shortcoming of the manifold model, we insist that realistic models of measurement should be pragmatic: we actually perform a finite number of observations and record a finite number of events. Thus, the conventional infinitude of events that we adopt to model classical spacetime structure seems to be a gross generalization of little operational value: we have no actual experience of a continuous infinity of events and their infinitesimal differential separation cannot be recorded in the laboratory.

However, due to the success of the classical model of observation at large scales one expects a connection between the realistic models and their

¹Algebra and Geometry Section, Department of Mathematics, University of Athens, Panepistimioupolis 157 84, Greece; e-mail: iraptis@eudoxos.dm.uoa.gr.

²Friedmann Laboratory for Theoretical Physics, SPb UEF, Griboyedova, 30–32, 191023, St. Petersburg, Russia; e-mail: gudrs@mail.ru.

ideal counterpart. The anticipated connection could be formulated in terms of a correspondence principle. That is to say, the structure of ideal observations arises in some kind of limit of the structures of pragmatic measurements. The aim of this paper is to provide a physical account for this correspondence.

The central object of the correspondence will be the *classical event* living in the limit manifold. This is modeled by a point there and a classical observer records no quantum interference of events. But in the pragmatic regime we interpret the event as a *pure state of spacetime* and we admit coherent quantum superposition between events. In this sense the quantum substratum of pragmatic events ‘decoheres’ to the classical point event of the limit manifold and the physical meaning of the correspondence principle is the usual quantum one due to Bohr.

We use the finitary substitutes proposed by Sorkin (1991) to model combinatorial relations between events in realistic measurements. The incidence algebras due to Rota (1968) are employed to accomplish the same thing, but operationally, that is, algebraically. There is a duality implicit here that is related to familiar notions about the duality of quantum dynamics. In our treatment the Sorkin model is held to represent an evolution of states much like the Schrödinger picture of quantum systems, while the Rota model recalls the evolution of operators similar to the Heisenberg picture. Our approach lies with the latter picture, although one is always able to switch back to the Sorkin scheme (Zapatin, 1998). In brief, we propose that posets describe the dynamical evolution of events when the algebras describe the dynamical evolution of event determinations (operations).

Our algebraic approach is constructive, that is, we provide a matrix representation for the algebras employed. The latter possess preferred elements that represent the pragmatic observations that in the ideal limit are expected to yield the irreducible elements of classical observations: the manifold point events. They constitute abelian subalgebras of the incidence algebras and are coined *stationaries*. Of course, actual pragmatic observations are expected to effect dynamical transitions between quantum states of spacetime (stationaries). These are modeled by noncommuting operations in the algebra of pragmatic events and are called *transients*. We anticipate that stationaries will correspond to classical point events in the limit manifold (in Section 5 we show how sequences of stationaries become stationery recording classical events), while transients correspond to some kind of tangent structure at an event.

To deal with the second shortcoming of the manifold model we note that at the pragmatic level of observations there are no points, but only algebras. We call this feature *alocality*. Nevertheless, we require the classical correspondence limit to yield the familiar local structure of spacetime: the point event and the space tangent to it. Since any point of the limit manifold

can be the host of a singularity of some important physical field, the alocal quantum pragmatic substratum may prove to be an effective resolution of spacetime into finite quantum elements. The quantum substratum is a-singular (finite) because a-local. Thus, from our perspective, locality, that is, the assumption of a differential continuum for spacetime (Einstein, 1924), is the prime reason for singularities, so that at the pragmatic level of observation we abandon it. We suggest a plausible quantum theory of spacetime structure with a strong operational and finitistic character. Then, based on the algebraic models of pragmatic observations we may develop a noncommutative differential geometry to erect a quantum theory of gravity on it (Parfionov and Zapatin, 1995). The correspondence limit suggested in the present paper may also be employed in this context to recover the classical algebra of spacetime observables and the conventional differential geometry of the spacetime manifold (used to describe general relativity) from the pragmatic noncommutative quantum substratum.

It should be mentioned that our algebraic approach is rather flexible in the following sense: alternatively to the novel notion of a-locality in the pragmatic regime we can formulate the notion of *nearest neighbor* connections. The latter was assumed by Finkelstein (1985) to be the principal characteristic of the physical causal topology in the quantum deep so as to localize in some sense a causality relation between events (Bombelli *et al.*, 1987). This causality relation was modeled by a partial order. Thus, if we physically interpret Sorkin's finitary substitutes as causal sets (Sorkin, 1995), a recent result (Zapatin, 1998) allows us to represent the 'nearest neighbor' causal connection between events algebraically in the pragmatic regime and thus vindicate Finkelstein's demand for an algebraic representation of immediate causality (Finkelstein, 1985). There, in turn, we have the advantage of interpreting this connection operationally and studying its quantum properties. The question we are confronted with is, What is the physical meaning in the pragmatic algebraic regime of the Sorkin–Finkelstein local causality? As an answer we expect a formulation of local causality in operational terms with a quantum interpretation, something that is missing in Sorkin's picture, which is dual to ours. Affine to this question is the following one: How can our pragmatic event determinations accord with and be adequate models of the causal structure of the world at small scales?

Finally, inspired by the Sorkin (1991) approach, we contend that pragmatic measurements can be subjected to refinements. In passing to the dual picture we deal with algebras and, in accordance with the correspondence limit, the ideal ultimate refinement corresponds to what is known as the algebra of classical observables (coordinates) and the manifold supporting them.

1. FINITARY PRELIMINARIES

A finitary topological space is defined in Sorkin (1991) as a space with any bounded region in it consisting of a finite number of points. This seems to be a reasonable model for actual measurements involving a finite number of events during experiments of finite spatiotemporal extent.

Any finitary topological space \mathcal{M} can be equivalently pictured as poset. Introduce the relation “ \rightarrow ” between points of \mathcal{M}

$$p \rightarrow q \Leftrightarrow \text{the constant sequence } \{p, p, \dots, p, \dots\} \text{ tends to } q$$

using the standard definition of convergence: a sequence $\{p_1, p_2, \dots\} \rightarrow q$ iff for any open set U containing q there is a number N_U such that $p_n \in U$ for any $n \geq N_U$.

The obtained relation “ \rightarrow ” is always reflexive ($p \rightarrow p$) and transitive ($p \rightarrow q, q \rightarrow r$ imply $p \rightarrow r$). Vice versa, any quasiordered set $(\mathcal{M}, \rightarrow)$ acquires a topology defined through the closure operator on subsets $P \subseteq \mathcal{M}$:

$$\mathbf{Cl}P = \{q: \exists p \in P p \rightarrow q\}$$

For technical reasons (see Section 2) we employ the Alexandrov (1956) construction of *nerves* to substitute the continuous topology. Recall that the nerve \mathcal{H} of a covering \mathcal{U} of a manifold \mathcal{M} is the simplicial complex whose vertices are the elements of \mathcal{U} and whose simplices are formed according to the following rule. A set of vertices (that is, elements of the covering) $\{U_0, \dots, U_k\}$ form a k -simplex of \mathcal{H} if and only if they have nonempty intersection:

$$\{U_0, \dots, U_k\} \in \mathcal{H} \Leftrightarrow U_0 \cap U_1 \cap \dots \cap U_k \neq \emptyset$$

Any nerve \mathcal{H} being a complex can be as well treated as a poset, denoted also by \mathcal{H} . The points of the poset \mathcal{H} are the simplices of the complex \mathcal{H} , and the arrows are drawn according to the rule

$$p \rightarrow q \Leftrightarrow p \text{ is a face of } q$$

In the nondegenerate cases the posets associated with nerves and those produced by Sorkin’s (1991) ‘equivalence algorithm’ are the same. We choose nerves because their specific algebraic structure makes it possible to build the dual algebraic theory.

2. INCIDENCE ALGEBRAS

The notion of incidence algebra of a poset was introduced by Rota (1968) in a purely combinatorial context. Let P be a poset. Consider the set of formal symbols $|p\rangle\langle q|$ for all $p, q \in P$ such that $p \leq q$ and its linear span

$$\Omega = \text{span}\{|p\rangle\langle q|\}_{p \leq q} \quad (1)$$

and endow it with the operation of multiplication

$$|p\rangle\langle q| \cdot |r\rangle\langle s| = |p\rangle\langle q|r\rangle\langle s| = \langle q|r\rangle \cdot |p\rangle\langle s| = \begin{cases} |p\rangle\langle s| & \text{if } q = r \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

The correctness of this definition of the product, that is, the existence of $|p\rangle\langle s|$ when $q = r$, is due to the transitivity of the partial order. The obtained algebra Ω with the product (2) is called *incidence algebra* of the poset P .

The incidence algebra Ω is obviously associative, but not commutative in general. Namely, it is commutative if and only if the poset P contains no arrows.

Let us split Ω into two subspaces

$$\Omega = \mathcal{A} \oplus \mathcal{R}$$

where

$$\mathcal{A} = \text{span}\{|p\rangle\langle p|\}_{p \in P} \quad (3)$$

and call

$$\mathcal{R} = \text{span}\{|p\rangle\langle q|\}_{p < q}$$

the *module of differentials* of the poset P . It is a fact that \mathcal{R} is a bimodule over \mathcal{A} .

As we refine the poset, the limit space is intended to be a manifold. The incidence algebras are dual objects to posets, therefore their behavior should be similar to that of differential forms in classical geometry. The algebra \mathcal{A} is intended to play the role of classical coordinates, while \mathcal{R} should be graded, being an analogue of the module of differential forms.

For this aim we consider only simplicial complexes, which are treated as posets. $p \leq q$ means that p is a face of q . The elements of simplicial complexes are naturally graded. Then any basic element $|p\rangle\langle q|$ of the incidence algebra Ω acquires a degree that is the difference of the degrees of its constituents:

$$\text{deg}|p\rangle\langle q| = \text{the difference of cardinalities of } p \text{ and } q \quad (4)$$

splitting Ω into linear subspaces

$$\Omega = \Omega^0 \oplus \Omega^1 \oplus \dots \quad (5)$$

with

$$\Omega^0 = \text{span}\{|p\rangle\langle p|\} = \mathcal{A}$$

...

$$\Omega^n = \text{span}\{|p\rangle\langle q|\}_{\text{deg}|p\rangle\langle q|=n}$$

...

making Ω a graded algebra:

$$\forall \omega \in \Omega^m, \quad \omega' \in \Omega^n, \quad \omega\omega' \in \Omega^{m+n}$$

and therefore making the module of differentials \mathcal{R} a graded \mathcal{A} -bimodule:

$$\mathcal{H} = \Omega^1 \oplus \Omega^2 \oplus \dots$$

This grading makes the incidence algebras discrete differential manifolds (Dimakis and Müller-Hoissen, 1999) as they possess both commutative scalars (the subalgebra \mathcal{A}) and differentials over it (the module \mathcal{R}). For a more detailed account the reader is referred to Breslav and Zapatrin (1999).

3. ROTA TOPOLOGY AND THE DUALITY

In this section we establish a duality between a certain class of finitary substitutes and their incidence algebras. We select this class in such a way that canonical mappings between the points admit conjugate homomorphisms of incidence algebras making the correspondence between posets and algebras functorial.

As it was shown in the previous section, its noncommutative incidence algebra can be associated with any poset. It was proved by Stanley (1968) that if two posets have isomorphic incidence algebras, then they are isomorphic. The reverse procedure building a poset $P(\Omega)$ from an arbitrary finite-dimensional algebra Ω was suggested in Zapatrin (1998). Let us briefly describe the construction.

The elements of the poset $P(\Omega)$ are the irreducible representations of the algebra Ω . Building the partial order on $P(\Omega)$ consists of two steps. First, the nearest neighbor connections $p \rightarrow q$ are built according to the following rule: let p, q be two irreducible representations of Ω ; denote by p^0, q^0 their kernels:

$$p^0 = p^{-1}(0); \quad q^0 = q^{-1}(0)$$

which are ideals in Ω . Then define the nearest neighbors $p \rightarrow q$:

$$p \rightarrow q \Leftrightarrow p^0 q^0 \neq p^0 \cap q^0 \tag{6}$$

where the left-hand side $p^0 q^0$ is understood as the product of subsets of Ω . The resulting partial order on the set $P(\Omega)$ is obtained as the transitive closure of the relation (6). The topology associated with this partial order is referred to as *Rota topology*.

When the algebra Ω is commutative, the Rota topology is discrete (no linked pairs $p \rightarrow q$). The obtained topology becomes nontrivial only when Ω is noncommutative.

Remark. When all irreducible representations of Ω are one-dimensional we can build two topologies on the set $P(\Omega)$, the Gel'fand and Rota ones, and it is interesting to compare them. The result is the following: the Gel'fand topology is always discrete, while the Rota topology may be nontrivial.

The possibility of mutual transitions between finitary topological spaces and algebras is based on the following theorem (Zapatrin, 1998):

If the algebra Ω is the incidence algebra $\Omega(P)$ of a poset P , then the resulting poset is isomorphic to P :

$$P \simeq P(\Omega(P))$$

As mentioned above, Stanley's (1968) theorem claims that

$$\Omega(P) \simeq \Omega(Q) \Leftrightarrow P \simeq Q$$

and one could expect that a poset homomorphism, that is, a continuous mapping of appropriate finitary topological spaces, should give rise to a homomorphism of their incidence algebras. Alas, this is not the case for general posets.

To gather functoriality we have to restrict somehow the class of posets we are dealing with and the mappings between them. We did this already in the previous section in order to make the incidence algebras graded. Namely, we restricted ourselves to simplicial complexes. To make incidence algebras dual objects, following Alesandrov (1956), we restrict the mappings between simplicial complexes to simplicial mappings only. Recall that a mapping $\omega: \mathcal{H}_\alpha \rightarrow \mathcal{H}_{\alpha'}$ between two simplicial complexes \mathcal{H}_α and $\mathcal{H}_{\alpha'}$ is said to be simplicial if:

- The ω -image of any vertex in \mathcal{H}_α is a vertex in $\mathcal{H}_{\alpha'}$

$$\omega(\mathcal{H}_\alpha^0) \subseteq \mathcal{H}_{\alpha'}^0 \quad (7)$$

- ω is completely defined by its values on the vertices of \mathcal{H}_α .
- ω preserves simplices.

Under this condition the correspondence between posets and their incidence algebras becomes functorial. With any $\omega: \mathcal{H}_\alpha \rightarrow \mathcal{H}_{\alpha'}$ its adjoint $\omega^*: \Omega(\mathcal{H}_{\alpha'}) \rightarrow \Omega(\mathcal{H}_\alpha)$ is defined in the following way. Let $|P\rangle\langle Q|$ be a basic element of $\Omega(\mathcal{H}_{\alpha'})$. Then Q may be represented as a disjoint sum $Q = P + S$ and we put

$$\omega^*(|P\rangle\langle Q|) = \sum\{|p\rangle\langle q|: \omega(p) = P; q = p + s, \omega(s) = S\} \quad (8)$$

The direct verification shows that the so-defined ω^* is always a homomorphism of the incidence algebras. As a result, we have the duality between simplicial complexes and their incidence algebras: if ω is surjective, its adjoint ω^* is injective and vice versa.

4. PHYSICAL INTERPRETATION OF INCIDENCE ALGEBRAS

The differential manifold model of spacetime is an ill-founded assumption and a gross generalization of what we actually experience as spacetime. It is essentially based on the nonoperational supposition that we can pack an infinity of events in an infinitesimal spacetime volume element when, in fact, we only record a finite number of them during experiments of finite duration in laboratories of finite size. It is exactly due to this trait of the manifold model that at small scales our theories of quantum spacetime structure and dynamics are plagued by nonrenormalizable infinities of the values of many important physical fields. On the other hand, the requirement that the laws of nature are local almost mandates the assumption of smoothness for spacetime and we seem to get back to square one. However, the success the manifold has enjoyed in picturing the local dynamics of matter should not mask the unphysicality of its character, especially at small scales. In particular, quantum theory, when applied to investigate the structure and dynamics of spacetime in the small, is simply incompatible with a classical, nonoperational ideal of a continuous infinity of events labeled by commutative coordinates. Pragmatic measurements of quantum spacetime are finite and inevitably induce uncontrollable dynamical perturbations to it. Thus, the requirement for operability and finiteness as well as the success that a quantum theory of matter has had when formulated algebraically motivate us to formulate an algebraic theory of pragmatic finite measurements of spacetime at quantum scales.

The local structure of the differential manifold is the point event and its infinitesimal differential neighbors in the tangent space. As mentioned above, this classical geometric structure serves us well in casting dynamical laws as differential equations (classical locality), but is rather inadequate for picturing actual quantum spacetime measurements that are finite, hence free from infinities (singularities). Especially in the quantum deep this classical conception of locality cannot survive. We propose to revise it by substituting the geometrical point events and the space of directions tangent to them by finitely generated algebras affording a cogent quantum spacetime interpretation for their structure. In this sense our scheme is local and finitary and more likely to evade the infinities of the differential manifold. Of course, the ‘naturalness’ of our substituting quantum alocality for the classical differential locality will be vindicated if we are able to recover the limit manifold

with its classical observables and differential structure by some kind of correspondence principle applied to the local algebras of pragmatic measurements. We carry this out in Section 5.

We give the following physical meaning to the elements of Ω in (3):

1. \mathcal{A} constitutes the space of stationaries. The latter can be thought of as elementary acts of determination of the pure states of quantum spacetime. We interpret them as quantum spacetime events. The algebraic connective ‘+’ between them is interpreted as coherent superposition between quantum events. The commutativity of stationaries foreshadows the compatibility of the determinations of the coordinates of events in the classical manifold regime (Section 5). In the dual (poset) picture the stationaries correspond to self-incidences $p \rightarrow p$.

2. Ω^1 constitutes the space of transients. These can be thought of as elementary quantum dynamical processes between stationaries; thus they represent discrete one-step transitions between quantum spacetime events. Transients do not commute with each other and this foreshadows the Lie structure of covectors in the limit space.

3. Ω^i ($i \geq 2$) constitute the spaces of paths which are thought of as composites of transients. If we associate with a transient a quantum of an additive physical quantity like energy (or its dual time), then the total grade of the appropriate element of the algebra corresponds to the total energy associated with it (or to the duration of the whole transition process).

In the Motivation we alluded to the Sorkin poset scheme as being an analog of the Schrödinger picture of quantum dynamics, while our algebraic approach is the simile of the Heisenberg picture: this is based on the duality of the two approaches (Section 3). In an analogous way quantum states are the linear duals of the operators in the conventional algebraic approach to quantum mechanics.

Here, too, any finitary substitute is associated with an incidence algebra in such a way that the topology of the poset is the same as that encoded in the algebra. This resembles the fact that the Schrödinger and the Heisenberg pictures encode the same information about quantum dynamics. Furthermore, the arrows between point events in the Sorkin scheme can be thought of as the directed dynamical transitions of spacetime event-states, while in our picture such dynamical connections are between pragmatic operations. The topology in both schemes is physically interpreted as dynamical connections between events although our picture, being algebraic, naturally affords a quantum interpretation.

5. LIMITING PROCEDURE AND THE CORRESPONDENCE PRINCIPLE

When spacetimes are substituted by finitary topological spaces, we may consider finer or coarser experiments. That is why we have to formalize the notion of refined experiment. Within the Sorkin discretization procedure (Section 1) a refinement means passing to an inscribed covering of the manifold. In this case any element of the finer covering is contained in an element of the coarser one. Since we are dealing with nerves and simplicial mappings between them we have to take care of the condition (7). Recall that a vertex of the nerve is associated with an element of the covering. In general it may happen that a small region of the fine covering can belong to two elements of a coarser one. So for any element of the fine covering we have to keep track of its origin in order for (7) to hold.

Each step of a limiting procedure, that is, a refined covering, gives rise to a projection of appropriate complexes: the finer one is projected to the coarser one. In the dual framework we have an injection of the smaller algebra associated with a coarser measurement to the bigger one.

In general, limiting procedures for approximating systems (whatever they be, posets or algebras) are organized using the notion of converging nets. Namely, each pragmatic observation is labeled by an index α and we have the relation of refinement $>$ on observations: $\alpha > \alpha'$ means that the observation α is a refinement of α' .

When we are dealing with posets with each pair α, α' such that $\alpha > \alpha'$ a canonical projection $\omega_{\alpha'}^{\alpha}: \mathcal{H}_{\alpha} \rightarrow \mathcal{H}_{\alpha'}$ is defined such that for any $\alpha > \alpha' > \alpha''$

$$\omega_{\alpha''}^{\alpha} = \omega_{\alpha'}^{\alpha'} \omega_{\alpha}^{\alpha'}$$

and we introduce the set of threads. A thread is a collection $\{t_{\alpha}\}$ of elements $t_{\alpha} \in \mathcal{H}_{\alpha}$ such that

$$t_{\alpha} = \omega_{\alpha}^{\alpha'} t_{\alpha'}$$

whenever $\alpha > \alpha'$. Denote by T the set of all threads.

The next step is to make T a topological space, which is done in a standard way (Aleksandrov, 1956): T is a subspace of the total Cartesian product

$$T_0 = \times_{\alpha} \mathcal{H}_{\alpha}$$

while each of \mathcal{H}_{α} is a topological space. Endow T_0 with the product Tikhonov topology, then T , being a subset of T_0 , becomes topological space. Finally we obtain the limit space X as the collection of all closed threads from T . This procedure is described in detail in Sorkin (1991).

The scheme for building limit algebras is exposed in Landi and Lizzi (1999). As mentioned above, with any pair $\alpha > \alpha'$ of pragmatic observations we have a canonical injection

$$\omega_\alpha^{*\alpha'}: \Omega(\mathcal{H}_{\alpha'}) \rightarrow \Omega(\mathcal{H}_\alpha)$$

Moreover, due to the requirement (7) the restriction of each ω^* on commutative subalgebras $\mathcal{A} = \Omega^0 \subseteq \Omega$ is well defined. Now we first consider the set of all sequences

$$\Omega = x_\alpha \Omega(\mathcal{H}_\alpha) = \{\{a_\alpha\} | a_\alpha \in \Omega(\mathcal{H}_\alpha)\}$$

and select the set of *converging* sequences in the following way. Note that \mathcal{A} is an algebra. Introduce a norm $\|\cdot\|_\alpha$ in each finite-dimensional algebra $\Omega(\mathcal{H}_\alpha)$; then a sequence $\{a_\alpha\}$ converges if and only if for any $\epsilon > 0$ there exists a filter \mathcal{F}_ϵ of indices α such that

$$\forall \alpha, \alpha' \in \mathcal{F}_\epsilon \quad \alpha > \alpha' \Rightarrow \|\omega_\alpha^{*\alpha'} a_\alpha - a_{\alpha'}\|_{\alpha'} < \epsilon$$

Since any element of the limit algebra is a net we may consider the coupling between the limit algebra and the limit space which consists of nets. The result of this coupling is a converging net of numbers whose limit is thought of as the value of an element of the limit algebra at a point of the limit space.

The Sorkin scheme recovers the manifold in the limit of refinements of finitary posets. Our dual picture aspires to the same in the limit of resolution of pragmatic event determinations. Since our algebraic scheme affords a quantum spacetime interpretation, this limit can be thought of as a correspondence principle linking the finitary quantum spacetime substrata with the smooth classical spacetime manifold. The alocal, algebraic quantum spacetime determinations of the substrata converge to the local geometric spacetime point and its cotangent space. This is to be contrasted, for instance, with the Bombelli *et al.* (1987) causal set scenario where the limiting procedure may be thought of as a ‘random sprinkling’ of events according to some appropriate distribution, so that the ‘limit spacetime manifold,’ with its topological, differential, and Lorentz-causal structure, arises as a statistical average of causal sets, and thus is essentially of *thermodynamic* nature.

On the other hand, our correspondence limit is well-defined in the *quantum* (rather than statistical) sense like the well-known correspondence principle: the pragmatic quantum stationaries decohere to the point events of the limit manifold, and the noncommuting transients to covectors.

6. CONCLUDING REMARKS

In the present paper we gave a quantum spacetime interpretation to the incidence algebras induced by posets, which, in turn, correspond to finitary

topological spaces. Sorkin's limit for recovering the manifold as a maximal refinement of finitary posets is cast here as Bohr's correspondence principle. Still, due to the implausibility of any notion of preexisting space in the quantum dynamical deep, we would rather give a more physical, causal, or temporal interpretation to the posets' partial order (Sorkin, 1995), so that we can link our algebraic scheme with Bombelli *et al.*'s (1987) causal set approach to quantum gravity. Our quantum interpretation of the incidence algebras induced by causal sets is a first step at yet another attempt at quantizing causality (Finkelstein, 1969). A previous result of Zapatrin (1998) and Finkelstein's (1985) claim that immediate causal links between events represent the physical causal topology motivated us to try to link the present work with causal sets. This project, however, is still at its birth.

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